Sometimes Less is More: Risk Aversion, Balanced Growth, and the (sub) Optimality of Entrepreneurial Insurance

Neville N. Jiang, University of Massachusetts Lowell
Alexandros Loukas, Washington University in St. Louis
Ping Wang, Washington University in St. Louis and NBER
Haibin Wu, City University of Hong Kong

March 2024

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Abstract

Is promoting entrepreneurship always conducive to long-run growth? To what extent should policymakers strive to ensure entrepreneurial risk away? We explore these questions within a highly tractable endogenous growth model with occupational choices, where individuals are heterogeneous in their risk attitude and entrepreneurial ability. Less risk-averse and sufficiently productive agents become entrepreneurs who create firms and contribute to growth by expanding product variety. More risk-averse and less productive agents become workers and contribute to growth by enhancing labor and capital formation. As risk tolerance leads to misallocation on the intensive margin, encouraging entrepreneurship may also hinder aggregate productivity. The interplay of these forces results in a non-monotone relationship between the rate of entrepreneurship and balanced growth. Decentralized equilibria entail suboptimal allocations with either too few or too many active producers, even in the absence of distortions or financial frictions. Ensuring some entrepreneurial risk away is almost always growth-enhancing, but it is never optimal to provide full insurance.

JEL codes: D50, E13, J24, L26, O41.

Keywords: Entrepreneurship, Occupational Choice, Risk Aversion, Endogenous TFP, Balanced Growth, Non-Monotone, Misallocation.

Correspondence: Ping Wang, Department of Economics, Washington University in St. Louis, St. Louis, MO 63130, U.S.A.; tel: 314-935-5632; e-mail: pingwang@wustl.edu.

1 Introduction

For over a century, at least since the pioneering work of Schumpeter (1911) and Knight (1921), the role of active entrepreneurship in fostering long-run economic growth has been emphasized by economists and taken into serious consideration by policymakers. Either implicitly or explicitly, conventional wisdom dictates that encouraging business formation and dynamism has a decidedly positive impact on economic growth; that is, "more is more" when it comes to entrepreneurship and the process of development.[1] Is this a theoretically robust prediction that public policies should *always* aim to accommodate?

Entrepreneurs also form a special occupational group in the sense that, despite its small size relative to an economy's labor force, it holds a disproportionately large share of total wealth and income.[2] Individual heterogeneity plays a crucial role in shaping occupational decisions, and Kihlstrom and Laffont (1979) emphasize three particularly influential aspects: entrepreneurial ability, wealth and access to capital markets, and risk attitude. There is indeed a large array of literature focusing on the first factor starting from the seminal articles by Lucas (1978) and Rosen (1981), and an even larger body of work focusing on the second factor stemming from the original contributions of Banerjee and Newman (1993), Aghion and Bolton (1997), and Lloyd-Ellis and Bernhardt (2000).[3] Importantly, the third factor—risk attitude—remains relatively unexplored in the modern literature despite its plausibility and empirical relevance; see the results of Caliendo et al. (2009) using an experimentally validated survey.

Is promoting entrepreneurship always conducive to long-run growth? To what extent should policymakers strive to ensure entrepreneurial risk away? We explore these questions by taking into account individuals' risk attitude and entrepreneurial ability within a dynamic general equilibrium framework with heterogeneous agents making occupational choices.

In the spirit of Lucas (1978) as well as more recent studies such as Ghatak et al. (2007) and Jiang et al. (2010), we study an economic environment with heterogeneous abilities and endogenous occupational decisions. In contrast to this strand of literature, however, we explicitly consider the influence of risk attitude and attempt to fill the knowledge gap in that regard. We begin by examining how the presence of risk aversion affects entry into entrepreneurship and aggregate TFP formation in dynamic general equilibrium, and then inquire whether the decentralized outcome is efficient. We further study the scope for insurance policy against entrepreneurial risk for enhancing long-run economic growth by comparing the outcome of full (actuarially fair) insurance of entrepreneurial risk with the (uninsured) decentralized equilibrium.

[1] A plethora of classic papers such as Baumol (1990) and Murphy et al. (1991) suggest that policies aimed at supporting productive (as opposed to rent-seeking) entrepreneurship will spur innovation and growth. At the same time, virtually all major international and intergovernmental institutions such as the World Bank, IMF, OECD, and UNCTAD are systematically advocating for policies and programs that encourage more inclusive entrepreneurship and promote small and medium-sized enterprises.

[2] As documented by Cagetti and De Nardi (2006), entrepreneurs defined as self-employed business owners account for only 7.6% of the U.S. population, but hold about one third of total net worth. Furthermore, according to Quadrini (2000) entrepreneurial households receive more than 20% of total U.S. income.

[3] As the literature on entrepreneurship and macroeconomics or economic theory in general is too extensive to even attempt to cover, see the excellent surveys of Quadrini (2009) and Parker (2018).

Specifically, we develop an overlapping-generations endogenous growth model with occupational choices, in which young agents who are heterogeneous in their risk attitude and idiosyncratic productivity choose whether to become workers or entrepreneurs. In a perfect-foresight environment without any capital market imperfections, young workers are assumed to be net fund suppliers, while entrepreneurs are net borrowers. A young entrepreneur endowed with an inalienable business idea can transform loanable funds into productive capital with some probability q. If succeeded, she can then combine capital and labor to generate an intermediate good of a distinct variety akin to her business idea. The final consumption good is produced by a representative competitive firm that aggregates the available basket of intermediate goods.

The central insights of the paper can be summarized in the following points. First, the link between the rate of entrepreneurship and long-run growth need not be positive or even monotone. Balanced growth depends nonlinearly on the number of active entrepreneurs in the economy, as well as on an aggregative measure of their productive capacity, therefore resulting in a non-monotone relationship. There are three opposing forces at play in general equilibrium. On the one hand, a higher number of people opting into entrepreneurship expands the variety of intermediate products, leading to increased production of the final good and subsequently higher growth. On the other hand, the ensuing reduction in the number of workers decreases both the aggregate supply of labor and of loanable funds, which in turn depresses capital formation and lowers economic growth. Apart from how many individuals become entrepreneurs, it is of first-order importance what type of individuals do so. We show that occupational choices induce an inverse association between risk tolerance and entrepreneurial talent at the margin; thus promoting entrepreneurship in a decentralized economy will hinder total factor productivity (TFP) as firm entry is accommodated by the lower parts of the ability distribution.

The ambiguous relationships (of both sign and magnitude) predicted by our theory can help rationalize an important empirical fact that may contradict conventional wisdom: increases in the the rate of entrepreneurship need not be positively associated with the rate of economic growth, e.g. Blanchflower (2000). Importantly, our findings suggest that public and/or industrial policies aimed at promoting entrepreneurship *unconditionally* need not be growth-enhancing. In the presence of insufficient loanable fund supply, reducing the number of workers/savers can also be harmful for growth even in the absence of distortions or financial frictions.

Second, the allocations in the decentralized equilibrium are almost surely suboptimal and result in output/income misallocation both on the extensive and intensive margins. Specifically, a competitive market economy features either over- or under-entrepreneurship (depending on parameter values and distributions) compared to the growth-maximizing solution of a constrained social planner. This occurs due to three reasons, none of which stems for frictions or distortions. Individuals will consistently undervalue the marginal benefit of becoming entrepreneurs via the variety effect, and they also undervalue the marginal cost of becoming entrepreneurs incurred through the loanable fund supply effect. At the same time, heterogeneity in risk aversion distorts the efficient pattern of occupational sorting and leads to lower aggregate TFP formation.

In the empirically plausible case where the capital income share is lower than the labor income share, an additional policy implication arises. Introducing an actuarially fair insurance market that eliminates entrepreneurial risk will indeed align private and social marginal benefits, but it will still fail to correct for the undervaluation of private marginal costs, thereby leading to excessive entrepreneurship rates. This finding provides a reasonable theoretical explanation for the empirical evidence documented by Astebro (2003), namely that potential entrepreneurs can be overly optimistic to invest in less lucrative projects.

This study has an additional theoretical ramification that is perhaps worth mentioning. Ever since the seminal work of Akerlof (1970), it is well understood that asymmetric information— with or without limited liability and/or financial frictions— can lead to inefficient outcomes in the market for entrepreneurial talent, which in turn deters aggregate investment and productivity, e.g., Stiglitz and Weiss (1981) and De Meza and Webb (1987). Our analysis provides an alternative explanation: in economies with risk-averse individuals and/or insufficient loanable fund supply, promoting entrepreneurship can be harmful to long-run growth and endogenous TFP formation even without any informational or credit market imperfections.

To further explore the quantitative implications of our framework, the model economy is calibrated to aggregate and establishment-level data for the U.S. and it is able to closely match all targeted moments without producing unconventional parameter values. In addition, it fits the full establishment size distribution together with the full employment distribution by size quite well, even though we are targeting only a single point from each distribution.

We then compute the decentralized equilibrium under full insurance against entrepreneurial risk as well as the planner's solution. Removing misallocation related to occupational choices leads to sizeable balanced growth gains of about 0.6% on an annualized basis under full risk insurance, and up to 0.7% per annum under the efficient allocation. The results also indicate that the U.S. entrepreneurship rate (as per our measurement) is lower than the optimal one, and in the case of full insurance would lead to far too many entrepreneurs. We also find that about 97% of income growth losses vis-à-vis the planner's solution is due to misallocation on the intensive margin caused by the presence of risk aversion: *who* becomes an entrepreneur is far more important for long-run growth than *how many* people do so. A crucial policy insight is that encouraging a small number of highly skilled individuals to operate firms would be more beneficial than incentivizing a larger number of less capable entrepreneurs to do so.

The rest of the paper is structured as follows. Section 2 provides a detailed exposition of the structure and components heterogeneous-agent economy, its . Section 3 contains the main results pertaining to decentralized competitive equilibria, along with sharp characterizations of endogenous quantities and factor prices. Section 4 presents a number of related results for centralized economies (planner's solutions) and offers insights into how the introduction of an actuarially fair market for entrepreneurial risk shapes occupational choices and the macroeconomy. Section 5 carries out a further characterization of balanced growth equilibria with respect to changes in key parameters. Section 6 calibrates the model economy to U.S. data and presents quantitative evidence in favor of the potentially large misallocation losses predicted by our theory. Section 7 briefly summarizes our main results and offers some concluding remarks.

2 An Endogenous Growth Model with Occupational Choice

In this section we delineate the environment of the model economy: a overlapping-generations endogenous growth model populated by heterogeneous agents making occupational choices and carrying out production plans. In Figure 1 we outline the basic structure of the economy, in which the timing of events is numerically ordered from 1 to 5.

2.1 Environment, Endowments, Preferences

There is a continuum of unit measure of two-period lived agents. After an initial old generation at time $t = -1$, the economy consists of an infinite sequence of two-period lived overlapping generations without any population growth. Individuals are *ex-ante* heterogeneous in their *risk attitude*, $\rho \in \mathcal{P}$, and *entrepreneurial ability/productivity*, $z \in \mathcal{Z}$. All young agents are endowed with a draw from a (non-singular) stationary joint distribution, $G(\rho, z)$, along with one unit of labor, and an idea i of designing a particular intermediate good. The sample space is the set $\Omega = (\mathcal{P} \times \mathcal{Z}) \subset \mathbb{R}^2_+$ generated by the joint support of the two state variables. We do not impose any restrictions on $G(\rho, z)$ apart from the natural assumption that it is continuous a.e.

An agent born in period t chooses to become a worker or an entrepreneur when young, and supplies one unit of labor inelastically to market activities. Individuals value consumption only when old and have no positive bequest motive. Their preferences are represented by,

$$U(c_{t+1}; \rho) = (c_{t+1})^{1/(1+\rho)}$$

where c_{t+1} is consumption of the final good in old age, and $\rho \geq 0$ is an index of risk aversion measuring agents' attitude towards intertemporal risk. Notice that this simple power utility form implies strictly increasing and concave cardinal utility for any positive $\rho < \infty$, and nests the case of risk-neutral preferences when $\rho = 0$.

A young worker of generation t supplies her entire labor endowment augmented by human capital h to an old entrepreneur of generation $t - 1$. Subsequently, she saves the entirety of her income for consumption in the second period of her lifetime.

A young entrepreneur of generation t borrows from a bank to transform the loan into capital subject to some uncertainty; each business project is bound to succeed with (constant) probability $q < 1$. The success rate q is the source of risk in the economy. If failed, she cannot produce and need not repay the loan under limited liability. If succeeded, she can then combine her capital with young workers to implement her idea and produce an intermediate good of a specific variety when old. As long as varieties are imperfect substitutes in the aggregation process, firm owners gain some pricing power due to the downward-sloping demand for each variety. With perfect capital markets, she will fully repay her debt and consume her remaining profit.

2.2 Production and Financial Markets

The economy consists of three sectors: a perfectly competitive final good sector, a monopolistically competitive intermediate goods sector, and a frictionless banking sector.

Intermediate goods sector. Intermediate producers (entrepreneurs) operate in a monopolistically competitive market and treat all prices but their own as given. Varieties are assumed to be imperfect substitutes in the final-good aggregation process thereby allowing producers to charge a fixed markup over their marginal cost, which in turn depends on their entrepreneurial ability/productivity (z). A young entrepreneur of generation t with a unique business idea and productivity z borrows $x_t(z)$ from a bank with a view to transforming the loan into productive capital, subject to an exogenous success rate $q < 1$,

$$k_{t+1}(z) = \begin{cases} x_t(z) & \text{with probability } q \\ 0 & \text{with probability } 1-q. \end{cases}$$

If succeeded, she hires young labor at $t+1$ to produce the intermediate good i according to the individual-specific technology,

$$y_{t+1}(z) = z\, k_{t+1}(z)^{\alpha} \left(\ell_{t+1}(z) h_{t+1}\right)^{1-\alpha} \tag{1}$$

The input of entrepreneurship is essential for production and higher entrepreneurial ability serves as Hicks-neutral technical progress, in the sense that the firm owner/manager is more efficient in combining the variable factors of production.

Final good sector. The final good sector is assumed to be perfectly competitive resulting in zero economic profit for the representative firm, so there is no need to specify its ownership structure. All intermediate goods provided by active entrepreneurs are aggregated into the production of a homogeneous consumption good – the numéraire, which is consumed by everyone in the economy – according to a standard CES technology,

$$Y_{t+1} = A \left(\iint\limits_{\mathcal{P}\times\mathcal{Z}} y_{t+1}(z)^{\theta}\, dG(\rho, z, \mathcal{E}) \right)^{\frac{1}{\theta}} \tag{2}$$

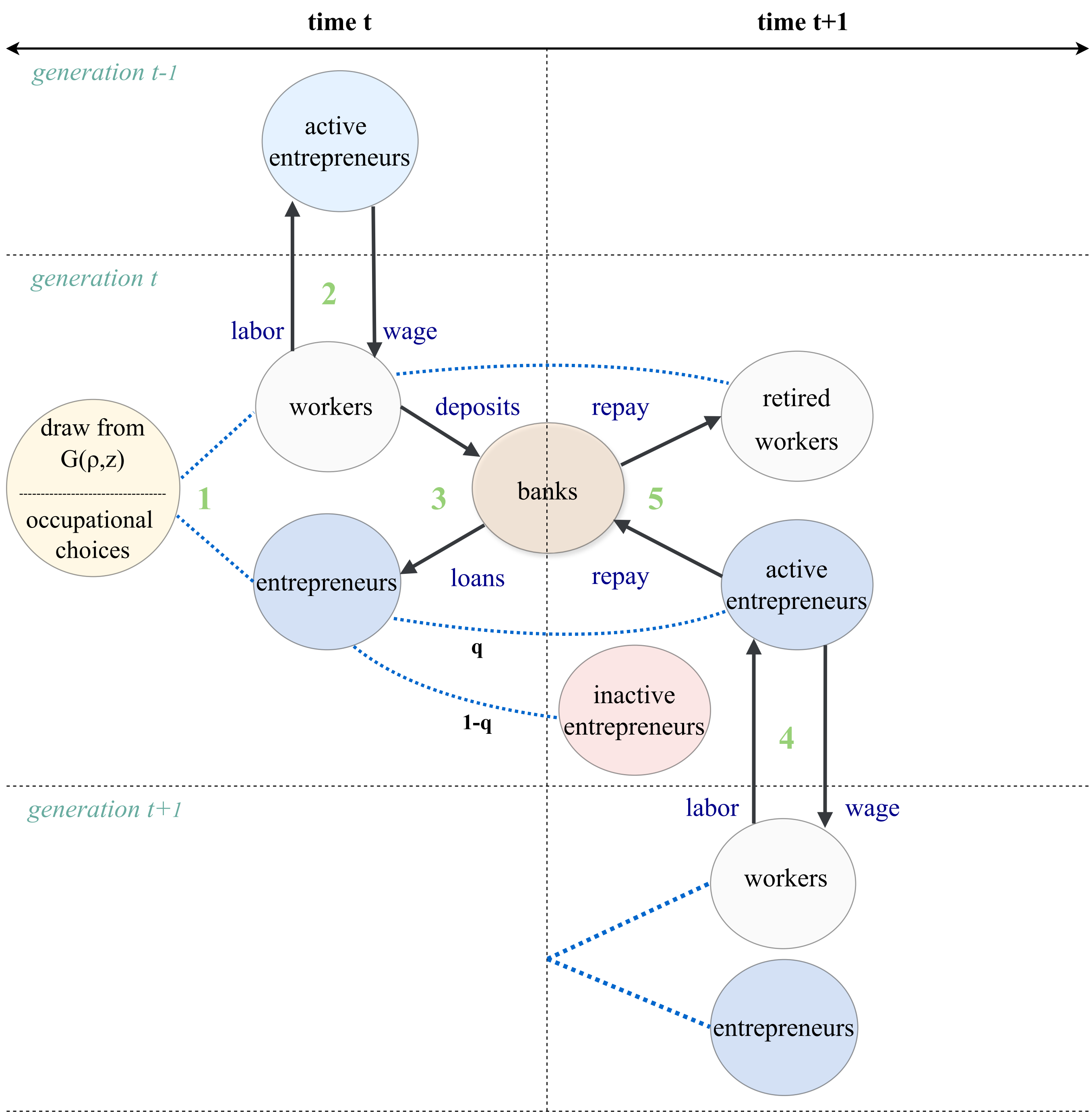
The solution to the cost minimization problem of the representative firm yields,

$$p_{t+1}(z) = A^{\theta} \left[\frac{y_{t+1}(z)}{Y_{t+1}} \right]^{-(1-\theta)} \tag{3}$$

which says that the price of intermediate good i produced with ability z, $p_{t+1}(z)$, is inversely related to the relative demand $\frac{y(z)}{Y}$, subject to a constant elasticity of demand equal to $-\frac{1}{1-\theta}$. We show below that the maximized level of output is strictly increasing in z, so more productive firms have lower marginal costs and can compete by charging a lower price for their product.

Financial market. The banking sector consists of financial intermediaries that receive deposits from workers and provide loans to potential entrepreneurs, without any operational costs. Under perfect capital markets and limited liability, the zero profit condition determines the (gross) loan rate (δ) to simply be a markup of the (gross) deposit rate (r_t), depending on the entrepreneurial success rate: $\delta_t = \frac{r_t}{q}$.

Figure 1: The Main Structure of the Model Economy



2.3 Entrepreneurs, Workers, and Occupational Choice

Entrepreneurs. The decision by an entrepreneur i of generation t at period $t+1$ is specified as follows. Given the effective wage rate w_{t+1}, the market loan rate δ_t, as well as quantities $k_{t+1}(z)$, Y_{t+1} and h_t, she determines her demand for labor by solving:

$$\max_{\ell_{t+1}(z)} \pi_{t+1}(z) = p_{t+1}(z)y_{t+1}(z) - w_{t+1}\ell_{t+1}(z)h_{t+1} - \delta_t k_{t+1}(z)$$
$$\text{subject to (3)}$$

The necessary and sufficient first-order condition implies,

$$\ell_{t+1}(z)h_{t+1} = \left[z^\theta \left(\theta A^\theta Y_{t+1}^{1-\theta}\right)\left(\frac{1-\alpha}{w_{t+1}}\right) k_{t+1}(z)^{\alpha\theta}\right]^{\frac{1}{1-(1-\alpha)\theta}} \tag{4}$$

where capital transformed when young has a positive effect on labor demand due to factor complementarity (in the Pareto sense). This relationship can be substituted into (1) and the profit function to derive,

$$y_{t+1}(z) = \left[z\left(\theta A^\theta Y_{t+1}^{1-\theta}\right)^{1-\alpha}\left(\frac{1-\alpha}{w_{t+1}}\right)^{1-\alpha} k_{t+1}(z)^{\alpha}\right]^{\frac{1}{1-(1-\alpha)\theta}} \tag{5}$$

$$\pi_{t+1}(z) = \Big[1-(1-\alpha)\theta\Big]\left[z^\theta\left(\theta A^\theta Y_{t+1}^{1-\theta}\right)\left(\frac{1-\alpha}{w_{t+1}}\right)^{(1-\alpha)\theta} k_{t+1}(z)^{\alpha\theta}\right]^{\frac{1}{1-(1-\alpha)\theta}} - \delta_t k_{t+1}(z) \tag{6}$$

We now go back one step to determine the entrepreneur's loan demand when young. Consider an entrepreneur i with risk attitude ρ whose optimization problem at this stage is,

$$\max_{k_{t+1}(z)} \mathbb{E}\left[U(c_{t+1})\right] = q\cdot\left(\pi_{t+1}(z)\right)^{1/(1+\rho)} + (1-q)\cdot 0 \tag{7}$$

We rearrange the first-order condition to obtain the loan demand function,

$$k_{t+1}(z) = \left[z^\theta\left(\theta A^\theta Y_{t+1}^{1-\theta}\right)\left(\frac{1-\alpha}{w_{t+1}}\right)^{(1-\alpha)\theta}\left(\frac{\alpha}{\delta_t}\right)^{1-(1-\alpha)\theta}\right]^{\frac{1}{1-\theta}} \tag{8}$$

and accordingly the labor demand function,

$$\ell_{t+1}(z)h_{t+1} = \left[z^\theta\left(\theta A^\theta Y_{t+1}^{1-\theta}\right)\left(\frac{1-\alpha}{w_{t+1}}\right)^{1-\alpha\theta}\left(\frac{\alpha}{\delta_t}\right)^{\alpha\theta}\right]^{\frac{1}{1-\theta}} \tag{9}$$

A higher aggregate demand for the final good or a lower labor cost raises profitability and thus loan demand, whereas an increase in the loan rate reduces it. Notice that $k_{t+1}(z)$ is independent of ρ, which means all entrepreneurs of the same generation with identical ability will borrow the same amount from banks. From (5), (6), and (8), the amount of intermediate good i produced and the corresponding profit become,

$$y_{t+1}(z) = \left[z \left(\theta A^{\theta} Y_{t+1}^{1-\theta}\right) \left(\frac{1-\alpha}{w_{t+1}}\right)^{1-\alpha} \left(\frac{\alpha}{\delta_t}\right)^{\alpha} \right]^{\frac{1}{1-\theta}} \tag{10}$$

$$\pi_{t+1}(z) = \frac{1-\theta}{\theta} \left(\frac{\delta_t}{\alpha}\right) k_{t+1}(z) \tag{11}$$

The expected utility of an entrepreneur of type (ρ, z) corresponds to,

$$\mathbb{E}[U^E(i, z, \rho)] = q \ (\pi(z)_{t+1})^{1/(1+\rho)} \tag{12}$$

which is clearly strictly decreasing in ρ, and strictly increasing in z since $\theta \in (0, 1)$.

Workers. Upon becoming a worker, an agent's decision is trivial: she works full time and deposits her entire wage income into a bank for consumption in old age. Her expected utility is,

$$\mathbb{E}\left[U^W(i)\right] = (r_t w_t h_t)^{1/(1+\rho)} = (q\delta_t w_t h_t)^{1/(1+\rho)} \tag{13}$$

where the deposit rate is $r_t = q\delta_t$ under the zero profit condition of the banking sector.

Occupational choice. The occupational decision between workers and entrepreneurs comes down to comparing the indirect utilities given by (12) and (13).

For a fixed level of risk aversion ρ, the ratio $\frac{\mathbb{E}[U^e]}{\mathbb{E}[U^w]}$ is continuous and strictly increasing in z from zero to infinity. Accordingly, for a fixed productivity level $z > \underline{z}$ where $\underline{z}$ ensures that the LHS is larger than the RHS for $\rho = 0$, the ratio $\frac{\mathbb{E}[U^e]}{\mathbb{E}[U^w]}$ is continuous and strictly decreasing in ρ, ranging from a constant greater than one to $q < 1$.[4] It follows that there exists a unique pair of critical values $\left(\rho^D(z), z^D(\rho)\right), \forall (\rho, z) \in \mathcal{P} \times \mathcal{Z}$, such that,

$$q(\pi_{t+1}(i, z^D))^{1/(1+\rho^D)} = (q\delta_t w_t h_t)^{1/(1+\rho^D)} \tag{14}$$

Individuals of joint type $(\rho < \rho(z)^D, z > z(\rho)^D)$ choose to be entrepreneurs each period. Apart from agents who are sufficiently productive and have a large absolute advantage in starting a firm, the economy will also feature a (potentially sizeable) mass of only moderately productive and less risk-averse agents. This is a source of misallocation in the economy, both on the intensive and extensive margin. A further characterization together with the determination of the measure of active entrepreneurs, N_{t+1}^d, is presented below in section 3.2.[5]

[4] The case $z < \underline{z}$ means that no agent with such z will choose entrepreneurship, as the cutoff for ρ is negative.
[5] Throughout the paper we use the superscript "D" to denote solutions in the decentralized economy.

2.4 Human Capital Accumulation

To close the model we need to specify the human capital accumulation process. As the focus of our paper is on risk attitude and entrepreneurship, there is no need for further complication on that front. We simply assume that the human capital stock evolves according to,

$$h_{t+1} = Y_t^{\beta} h_t^{1-\beta} \tag{15}$$

That is, total output of the final product is used as a proxy for aggregate current activity, which in turn contributes to the accumulation of human capital. Since real GDP is taken as given by entrepreneurs, the analysis is substantially simplified.

3 Competitive Equilibrium

We are now ready to study the equilibrium in the decentralized economy. We start by obtaining the loan and labor market clearing conditions. There is no need to consider the goods market separately as it automatically clears by virtue of Walras's law.

3.1 Market Clearing Conditions

The total demand for loans is simply the integral across all entrepreneurial loan demands, which can be also expressed as the measure of entrepreneurs (in the labor force) times the average amount of loan demanded within entrepreneurs. The total supply of loans comes from the aggregate wage income of workers. The loan market clearing condition is specified as,

$$\begin{aligned} \iint\limits_{\mathcal{P}\times\mathcal{Z}} k_{t+1}(z)\, dG(\rho, z, \mathcal{E}) &= \iint\limits_{\mathcal{P}\times\mathcal{Z}} w_t h_t \, dG(\rho, z, \mathcal{W}) \\ N_{t+1}^D \bar{k}_{t+1} &= (1 - N_{t+1}^D) w_t h_t, \forall t \geq 0 \end{aligned} \tag{16}$$

where $\bar{k}_{t+1} = \iint k_{t+1}(z)\, dG(\rho, z|\mathcal{E})$ is the *average* firm capital.[6] Similarly for the demand for labor, with the difference that only those entrepreneurs who succeeded in transforming loans into capital goods (qN_t^D) can hire labor to undertake production. The labor market clearing condition thus becomes,

$$\begin{aligned} q \iint\limits_{\mathcal{P}\times\mathcal{Z}} \ell_t(z)\, h_t \, dG(\rho, z, \mathcal{E}) &= \iint\limits_{\mathcal{P}\times\mathcal{Z}} h_t \, dG(\rho, z, \mathcal{W}) \\ qN_t^D \bar{\ell}_t &= 1 - N_{t+1}^D, \forall t \geq 0. \end{aligned} \tag{17}$$

where $\bar{\ell}_t$ is the *average* firm size by employment. Note that the time subscript on the RHS is $t+1$ because the number of entrepreneurs in the next period is determined by occupational choice in

[6] Average quantities are obtained by integrating with respect to the conditional distribution after occupational choices have been made, i.e., $\mathcal{E} := \{(\rho_i, z_j) \in \mathcal{P} \times \mathcal{Z} : \big(\rho_i < \rho^D(z_j)\big) \wedge \big(z_j > z^D(\rho_i)\big)\}$.

the current period. We continue by defining the concept of a dynamic competitive equilibrium.

Definition. *Given a non-singular joint distribution of risk attitude and entrepreneurial productivity,* $G(\rho, z)$, initial stock of human capital, h_0, *the measure of initial successful entrepreneurs* qN_0, *and the initial average amount of capital they hire,* $(\bar{k}_0)$, *a **dynamic competitive equilibrium** is a collection of quantity sequences* $\{Y_t, \bar{\ell}_t, \bar{k}_{t+1}, x_t, h_{t+1}, N^D_{t+1}\}_{t=0}^{\infty}$, *and a collection of price sequences* $\{w_t, r_t, p_t\}_{t=0}^{\infty}$, *such that:*

1. *given prices and endowments, every agent maximizes her expected utility for all* $t \geq 0$;
2. *an agent of type* (ρ, z) *born in period* t *chooses to become an entrepreneur if and only if* $\rho \leq \rho^D_t(z)$ and $tau \leq \rho^D_t(z)$, where ρ^D_t *is determined by* (14)
3. *the measure of entrepreneurs is* $N^D_{t+1} = \int_0^{\tilde{\rho}^D_t} \int_{\tilde{z}^D_t}^{\infty} dG(\rho, z)$, *and only a fraction* q *of them succeed;*
4. *human capital evolves according to (15) for all* $t \geq 0$;
5. *the labor, capital, and goods markets clear at all* $t \geq 0$

We thereby focus on perfect-foresight ***balanced growth equilibria*** in which the real variables, $Y, \bar{k}$, and h, all grow at constant rates, and N^d, $\bar{\ell}$, p, w,, r, $\tilde{\rho}^D$, and $\tilde{z}^D$, are all constant.

3.2 Occupational Choice

We are now ready to obtain the cutoff level for each pair of risk aversion/productivity that completely determines each agent's occupational choice. As shown below, $\rho^D_t(z)$ is unique and time invariant for each agent of type $(\rho, z) \in \mathcal{P} \times \mathcal{Z}$. By utilizing $\pi_{t+1}(z)$ in (11), the occupational choice condition (14) reads,

$$q^{\rho^D} \frac{1-\theta}{\alpha\theta} k_{t+1}(z) = w_t h_t, \ \forall t \tag{18}$$

which can be further combined with (8) and (16) to yield,

$$q^{\rho^D} \frac{1-\theta}{\alpha\theta} z^{\frac{\theta}{1-\theta}} = \frac{N^D_{t+1}}{1-N^D_{t+1}} \mathbb{E}_t \left[z^{\frac{\theta}{1-\theta}} | \mathcal{E} \right], \ \forall t \tag{19}$$

where the expectation is taken w.r.t. the joint distribution conditional on agents being entrepreneurs, $G(\rho, z|\mathcal{E})$. As shown in Figure 2, since the LHS of (19) is strictly decreasing from $\frac{1-\theta}{\alpha\theta}$ to 0 and the RHS is strictly increasing in ρ from 0 to infinity, ρ^D_t is uniquely determined and time-invariant. Thus, the number of entrepreneurs does not change over time.

3.3 Dynamics and the Balanced Growth Path

Since the number of people who choose to become entrepreneurs in each generation is the same, it turns out that the dynamics of this economy hinge solely on the average physical-to-human capital ratio. To express total output in terms of $(\frac{\bar{k}_t}{h_t})$ for any $t \geq 0$, start by integrating across all individual production plans in the final good production function,

$$Y_t = A\left(qN^d\right)^{\frac{1}{\theta}} \left(\iint\limits_{\mathcal{T}_\mathcal{E}\times\mathcal{Z}_\mathcal{E}} z^{\frac{\theta}{1-\theta}}\, dG(\rho, z|\mathcal{E})\right)^{\frac{1}{\theta}} \left[\left(\theta A^\theta Y_{t+1}^{1-\theta}\right)\left(\frac{1-\alpha}{w_{t+1}}\right)^{1-\alpha}\left(\frac{\alpha}{\delta_t}\right)^{\alpha}\right]^{\frac{1}{1-\theta}}$$

By manipulating the above equation and using the labor market clearing condition, we can express total output per unit of human capital as[7]

$$\begin{aligned} Y_t &= A\left(\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}\right]\right)^{\frac{1-\theta}{\theta}} \left(qN^d\right)^{\frac{1}{\theta}} \left(\bar{\ell}_t\, h_t\right)^{1-\alpha}\left(\bar{k}_t\right)^{\alpha} \\ \frac{Y_t}{h_t} &= A\,\mathcal{M}_\vartheta \left(qN^d\right)^{\frac{1}{\theta}} \left(\frac{1-N^d}{qN^d}\right)^{1-\alpha}\left(\frac{\bar{k}_t}{h_t}\right)^{\alpha} \end{aligned} \tag{20}$$

Apart from the exogenous level term A, the Solow residual is no longer a measure of our ignorance in this economy. Specifically, it consists of two endogenous quantities: a composite extensive-margin term shaped by the measure of entrepreneurs and workers; and an intensive-margin/TFP term, $\mathcal{M}_\vartheta := \left(\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}\right]\right)^{\frac{1-\theta}{\theta}}$, which is in fact a Hölder mean (also known as generalized mean) with exponent $\vartheta := \frac{\theta}{1-\theta}$ of active entrepreneurs' productivities, weighted by the conditional joint distribution of (ρ, z) on the support $\mathcal{T}_\mathcal{E} \times \mathcal{Z}_\mathcal{E}$. These are indeed key quantities, as they highlight the sectoral and distributional consequences of occupational choice on aggregate productivity. Put simply, total income is determined both by how many entrepreneurs are active in the economy, as well as by what type of entrepreneurs they actually are.

The next step is to show how the equilibrium wage rate is related to the ratio of the two state variables, k and h. Equation (20) can be combined with (4) and (17) to derive the market-clearing effective wage rate

$$\begin{aligned} w_t &= A^\theta \theta\,(1-\alpha)\,\left(\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}\right]\right)^{1-\theta}\left(\frac{1-N^d}{qN^d}\right)^{-1+(1-\alpha)\theta}\left(\frac{Y_t}{h_t}\right)^{1-\theta}\left(\frac{\bar{k}_t}{h_t}\right)^{\alpha\theta} \\ &= A\,\theta\,(1-\alpha)\,\mathcal{M}_\vartheta\left(qN^d\right)^{\frac{1-\theta}{\theta}}\left(\frac{1-N^d}{qN^d}\right)^{-\alpha}\left(\frac{\bar{k}_t}{h_t}\right)^{\alpha} \end{aligned} \tag{21}$$

and following a similar procedure we can derive the market-clearing deposit/loan rate

$$\delta_t = r_t/q = A\,\theta\,\alpha\,\mathcal{M}_\vartheta\left(qN^d\right)^{\frac{1-\theta}{\theta}}\left(\frac{1-N^d}{qN^d}\right)^{1-\alpha}\left(\frac{\bar{k}_t}{h_t}\right)^{\alpha-1} \tag{22}$$

Turning to the dynamics of the economy, the task is to analyze the path of the ratio of the two states. Total wage earnings at period t will be loaned to generation-t entrepreneurs. From

[7] Throughout the derivations, it is clear that we can switch from multiple to repeated integration by the Fubini-Tonelli theorem, given the σ-finiteness of probability spaces and measurability of functions.

the loan market clearing condition we have that

$$\frac{\bar{k}_{t+1}}{h_{t+1}} = \frac{1-N^d}{N^d}\frac{h_t}{h_{t+1}} w_t \tag{23}$$

After substituting in the human capital evolution equation (15) together with (20) and (21), the dynamic ratio of average physical to human capital becomes

$$\begin{aligned}\frac{\bar{k}_{t+1}}{h_{t+1}} &= \frac{1-N^d}{N^d}\left(\frac{Y_t}{h_t}\right)^{-\beta} w_t \\ &= \left[A^{1-\beta}\,\theta\,(1-\alpha)\,\mathcal{M}_{\vartheta}^{1-\beta}\left(qN^d\right)^{\frac{1-\theta-\beta}{\theta}}\left(\frac{1-N^d}{N^d}\right)\left(\frac{1-N^d}{qN^d}\right)^{-\alpha-\beta+\alpha\beta}\right]\left(\frac{\bar{k}_t}{h_t}\right)^{\alpha(1-\beta)}\end{aligned}$$

It follows from the above equation that, for any given initial conditions, $\frac{\bar{k}_t}{h_t}$ will converge in finite time to its balanced growth value

$$\left(\frac{\bar{k}}{h}\right)^{BGP} = \left[A^{1-\beta}\theta\,(1-\alpha)\,\mathcal{M}_{\vartheta}^{1-\beta}\left(qN^d\right)^{\frac{1-\theta-\beta}{\theta}}\left(\frac{1-N^d}{N^d}\right)\left(\frac{1-N^d}{qN^d}\right)^{-\alpha-\beta+\alpha\beta}\right]^{1/(1-\alpha+\alpha\beta)} \tag{24}$$

Once $(\frac{\bar{k}}{h})^{BGP}$ is reached, the system is on the balanced growth path where Y_t, h_t and k_t all grow at the same constant rate g^D:

$$\begin{aligned}1+g^D &= \frac{h_{t+1}}{h_t} = \left(\frac{Y_t}{h_t}\right)^{\beta} \\ 1+g^D &= \left[A\left(q\theta(1-\alpha)\right)^{\alpha}\mathcal{M}_{\vartheta}\left(qN^d\right)^{\frac{1-\theta}{\theta}}(1-N^d)^{1-\alpha}\right]^{\beta/(1-\alpha+\alpha\beta)}\end{aligned} \tag{25}$$

We can thus conclude that any competitive balanced-growth equilibrium features a non-monotone relationship between economic growth and the rate of entrepreneurship. In addition, the attained growth rate is suboptimal with probability one. We summarize these findings below.

Proposition 1. *In a decentralized equilibrium, encouraging entrepreneurship may or may not promote balancex growth. Moreover, if the joint distribution $G(\rho, z)$ is strictly monotone on all measurable sets of Ω, the attained balanced growth rate is suboptimal almost surely. Specifically,*

$$\frac{dg^D}{dN^d} \propto \underbrace{\frac{1-\theta}{\theta}\frac{1}{N^d}}_{\text{variety effect}} - \underbrace{\frac{1-\alpha}{1-N^d}}_{\substack{\text{loanable fund}\\ \text{supply effect}}} + \underbrace{\frac{1-\theta}{\theta}\frac{\frac{\partial}{\partial N^d}\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^d\right]}{\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^d\right]}}_{\text{TFP effect}} \gtrless 0 \quad a.s. \tag{26}$$

In the heart of Proposition 1 lies the fact that increasing the number of entrepreneurs gives rise to three opposing effects, as suggested by (26). On the one hand, more entrepreneurs means that greater intermediate product variety can be achieved, whose importance is gauged by $\frac{1-\theta}{\theta}$. We call this the *variety effect.* On the other hand, more entrepreneurs means less workers, and since workers are net savers, capital formation is being reduced. The labor income share $(1-\alpha)$ measures the importance of the *loanable fund supply effect.* Furthermore, the heterogeneity in (ρ, z) shapes the average productivity of active entrepreneurs in the economy increasing N^d induces a negative *TFP effect.* We have already shown that occupational choices result in a positive relationship between risk aversion and entrepreneurial ability, hence the decentralized allocation will necessarily involve a potentially large mass of less risk-averse agents with lower productivity. Put differently, due to risk aversion, $\mathcal{M}_\vartheta$ will not be (conditionally) maximal almost surely, even if the variety and loanable fund supply effects exactly offset each other. thereby deviating from the (constrained) optimal balanced growth rate.

4 Centralized Economy and Insurance Markets

In this section we analyze the differences between the number of entrepreneurs, endogenous TFP, and the balanced growth rate attained in the decentralized equilibrium and those obtained in a centralized economy. In addition, we examine the long-run implications of an actuarially fair insurance market for entrepreneurial risk.

4.1 A Constrained Central Planning Problem

Consider a central planner who wishes to maximize the long-run growth rate of the economy. The choices consist of a sequence of allocations $\{k_t^C(z), \ell_t^C(z)\}_{t=0}^{\infty}$, together a time-invariant measure N^c and the set of entrepreneurs $\mathcal{E}^c$, under the constraint that the saving rate is equal to the worker income share, $\theta(1-\alpha)$, as in the decentralized economy.[8] The latter condition constitutes a realistic perspective. If there is a government/institutional policy that allows the decentralized economy to achieve (or bring it closer to) the constrained planner's allocation, then it is the preferred policy choice. In such a case, what N^c and $\mathcal{E}^c$ would the planner pick, and how do they relate to N^d and $\mathcal{E}^d$?

The problem is solved in two stages. First, the planner chooses the level of capital $k_t^C(z)$ and labor $\ell_t^C(z)$ to be hired by each firm operated by z-type agents, keeping N^c and $\mathcal{E}^c$ fixed; second, he effectively picks N^c and $\mathcal{E}^c$, to satisfy the optimality condition for the balanced growth rate. Since the planning problem involves the maximization of a value *functional* over a suitable Banach space, it is accordingly formulated as a variational calculus program:

[8] Note that although the labor share of aggregate income is $(1-\alpha)$, this accounting identity attributes a "labor"' income component to entrepreneurial profits, which is a fraction $(1-\theta)$ of total income. Therefore, the worker share of income is $\theta(1-\alpha)$.

$$\max_{\substack{k_t(z)\ell_t(z),\\ N^c(\rho,z),\mathcal{E}^c(\rho,z)}} A^{\beta}\left(\iint_{\mathcal{P}\times\mathcal{Z}} z^{\theta}\left(k_{t+1}(z)/h_t\right)^{\alpha\theta}\ell_{t+1}(z)^{(1-\alpha)\theta}\,dG(\rho,z,\mathcal{E}^c)\right)^{\frac{\beta}{\theta}} \tag{27}$$

$$\text{subject to} \iint_{\mathcal{P}\times\mathcal{Z}} k_{t+1}(z)\,dG(\rho,z,\mathcal{E}^c)=\theta(1-\alpha)Y_t \tag{28}$$

$$q\iint_{\mathcal{P}\times\mathcal{Z}} \ell_{t+1}(z)\,dG(\rho,z,\mathcal{E}^c)=1-N^c \tag{29}$$

Proposition 2. *In the centralized economy, the optimal balanced growth rate corresponds to*

$$1+g^C=\left[A\left(q\theta(1-\alpha)\right)^{\alpha}\mathcal{M}_{\vartheta}^{C}\left(qN^c\right)^{\frac{1-\theta}{\theta}}(1-N^c)^{1-\alpha}\right]^{\beta/(1-\alpha+\alpha\beta)} \tag{30}$$

where the optimal number of entrepreneurs satisfies

$$\underbrace{\frac{1-\theta}{\theta}\frac{1}{N^c}}_{\text{variety effect}}-\underbrace{\frac{1-\alpha}{1-N^c}}_{\substack{\text{loanable fund}\\ \text{supply effect}}}+\underbrace{\frac{1-\theta}{\theta}\frac{\frac{\partial}{\partial N^c}\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^c\right]}{\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^c\right]}}_{\substack{\text{TFP effect}\\ (<0)}}=0\quad a.s. \tag{31}$$

$$N^c=\frac{\Phi^c(1-\theta)}{(1-\alpha)\theta+\Phi^c(1-\theta)},\quad \Phi^c:=\left(\frac{\overset{*}{z}}{\mathcal{M}_{\vartheta}^{c}}\right)^{\frac{\theta}{1-\theta}} \tag{32}$$

This condition coincides with that in (26), in the sense that the central decision maker chooses N^c so that the variety effect and the loanable fund supply effect are equally important at the margin. In addition, to maximize the TFP effect, the planner will choose the most able entrepreneurs (highest z) conditional on their total number being N^c.

In the decentralized equilibrium, why does N^d fail to satisfy the optimality condition? To ease the comparison we can rewrite (19) as

$$\frac{q^{\rho^D}(1-\theta)}{\theta}\frac{1}{N^d}=\frac{\alpha}{1-N^d} \tag{33}$$

$$N^d=\frac{q^{\hat{\rho}}\Phi_j^d(1-\theta)}{\alpha\theta+\Phi_j^d(1-\theta)},\quad \Phi_j^d:=\left(\frac{\hat{z}_j}{\mathcal{M}_{\vartheta}}\right)^{\frac{\theta}{1-\theta}} \tag{34}$$

We see that it differs from (**??**) on both sides of the equation. First, on the LHS, since agents are risk-averse, individual decision-makers will discount the importance of the variety effect. As a result, there will be "too less" entrepreneurship in the decentralized equilibrium (N^d tends to be smaller than N^c). However, the RHS is also different, which suggests that even if there is an actuarially fair market for entrepreneurial risk, the number of entrepreneurs would still not be optimal. This is the issue which we now turn to.

4.2 Actuarially Fair Insurance Market

Suppose there exists an *actuarially fair* insurance market for entrepreneurial risk in the decentralized economy: a large number of competitive risk-neutral insurance companies are willing to insure potential producers against the risk of failing to transform their loans into productive capital. In equilibrium, the zero profit condition leads to an insurance price equal to $p^I = 1 - q$, and all entrepreneurs will choose to be perfectly insured.

The occupational choice condition in that case can be expressed as if agents were risk-neutral,

$$\pi_{t+1}(z) = \delta_t w_t h_t \tag{35}$$

Then the number of entrepreneurs in the decentralized economy with full insurance satisfies,

$$\frac{1-\theta}{\alpha\theta}\,\bar{z}^{\frac{\theta}{1-\theta}} = \frac{N^{FI}}{1-N^{FI}}\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^{FI}\right] \tag{36}$$

Since the above is independent of ρ, the cutoff $\bar{z}$ is unique and applies to every agent in the economy. We are now ready to compare the rate of entrepreneurship rate in the decentralized equilibrium without insurance to that of the decentralized economy with full insurance, as well as to the constrained-first-best allocation in the centralized economy.

Proposition 3. *The equilibrium allocations in a decentralized economy with actuarially fair insurance markets, and thus full insurance against entrepreneurial risk, results in more entrepreneurs than one without such a market:* $N^{FI} > N^d$ *a.s.*

By eliminating entrepreneurial risk, full insurance always encourages entrepreneurship. However, observe that the importance of the loanable fund effect from the individuals' viewpoint amounts to α, not $(1-\alpha)$. The reason for this distortion is an intergenerational externality: becoming a worker contributes to current total output, which in turn raises the human capital of the next generation. The current generation internalizes only part of this externality since a higher level of human capital complements physical capital owned by current-period workers, thus individual decision-makers value the importance of the loanable fund supply effect by α instead of $1-\alpha$. Interestingly, the prevailing level of the capital income share is necessary and sufficient in determining whether N^{FI} will be higher or lower than N^c

Proposition 4. *The equilibrium allocations in a decentralized economy with actuarially fair insurance markets, i.e. full insurance against entrepreneurial risk, results in more (less) entrepreneurs than the centralized economy, if and only if* α *is less (greater) than* $1/2$*:*
$N^{FI} > N^c \iff \alpha < 1/2$ *a.s.;* $N^{FI} < N^c \iff \alpha > 1/2$ *a.s.*

An interesting case is when $N^d < N^c < N^{FI}$. In that case we can combine Proposition 1 and Propositions 1 and 2 to show that providing *some* insurance to entrepreneurial risk in a decentralized economy without an insurance market can be growth-enhancing, whereas providing *too much* insurance can be growth-retarding.

Proposition 5. *It is never optimal to provide full insurance. Moreover, if $\alpha < 1/2$,*

(i) *when the decentralized economy features less entrepreneurs than the centralized economy, i.e., $N^d < N^c$, providing some insurance to entrepreneurial risk in a decentralized economy without an insurance market is growth-enhancing;*

(ii) *when the decentralized economy features more entrepreneurs than the centralized economy, i.e., $N^d > N^c$, any provision of insurance to entrepreneurial risk in a decentralized economy without an insurance market is growth-retarding.*

Astebro (2003) provides empirical evidence that potential entrepreneurs can be overly optimistic to invest in less-desirable projects. In our paper, a full insurance that removes entrepreneurial risk can align private and social marginal benefits but fail to correct the undervaluation of the private marginal costs. When the capital income share is less than the labor income share, the decentralized equilibrium under full insurance features too much entrepreneurship. Thus, too much insurance results in too much optimism. Our finding therefore offers a plausible theoretical explanation for the empirical evidence specified above.

5 Further Characterization of the Balanced Growth Equilibrium

The results up to now allow us to investigate how the cutoff degree of risk-aversion ρ^D and the equilibrium number of entrepreneurs N^d respond to a change in the probability of success q or in the marginal distribution of risk-attitude, $F(\rho)$.

5.1 Changes in the Probability of Success

Consider an increase in the probability of entrepreneurial success (q). A greater q will raise the RHS of (19) without affecting the LHS. The result is obvious: a greater q leads to a higher cutoff degree of risk-aversion and more entrepreneurs. This result is also intuitive: a greater chance of success makes entrepreneurs a more attractive occupation.

Will a higher q raise the balanced growth rate? Intuitively, raising q should be growth-enhancing for two reasons. First, a higher fraction of the savings can be transformed into productive physical capital. Second, a greater variety of the intermediate goods can be produced under a higher q and the variety effect is growth-enhancing. We can see these two direct effects from the terms (q^α) and $(qN^d)^{\frac{1-\theta}{\theta}}$ in (**??**), respectively. There, however, is an indirect effect through changing the number of entrepreneurs. As shown in Proposition 1, there is no definite relationship between number of entrepreneurs and growth. If there are already too many entrepreneurs in the decentralized economy, more entrepreneurs resulting from a higher q will further lower the growth rate. Taking both the direct and the indirect effects into account, a higher q may or may not lead to a higher growth rate. In summary, we have:

Proposition 6. *An increase in the entrepreneurial probability of success q leads to higher threshold degrees of risk aversion ρ^D, and thus more entrepreneurs N^d. Its effect on the balanced growth rate is, however, ambiguous.*

Since the effect of a higher q on the growth rate is ambiguous and since a higher q unambiguously lowers the financial markup ($\frac{\delta-r}{\delta} = 1 - \frac{r}{\delta} = 1 - q$), our model predicts an ambiguous relationship between the financial markup and the growth rate in response to changes in the underlying source of risk in the economy.

5.2 Changes in the Risk-Attitude Distribution

We are particularly interested in changes in the risk-attitude distribution in two specific ways: the new distribution is a mean-preserving spread of F and the new distribution first-order stochastically dominates F. Before we go on, it should be noted that unlike changing q, changing F does not have directly observables effects on the balanced growth rate; it does have, nevertheless, an indirect impact through occupational choices and general equilibrium effects.

Exercise 1: First-Order Stochastic Dominance $(F \to F^{FOSD})$

Under distribution F^{FOSD} which first-order stochastically dominates F, the LHS of (19) will be lowered for any given ρ. Once again, our intuition is confirmed: If agents in an economy become more risk averse, there will be fewer agents choosing to become entrepreneurs and the the cutoff degree of risk-aversion will become higher.

Exercise 2: Mean-Preserving Spread $(F \to F^{MPS})$

Since F^{MPS} is a mean-preserving spread of F, $F^{MPS}(\rho)$ is greater (smaller) than $F(\rho)$ all for ρ smaller (greater) than μ_ρ– the mean. Consequently, the graph of function $\Psi(N) \equiv \frac{N}{1-N}$ under F^{MPS} crosses that under F from above at ρ^* as in Figures 6.a and 6.b. The effect of a mean-preserving spread on the equilibrium number of entrepreneurs turns out depending on whether the cutoff degree is greater than or smaller than the mean. Think a mean-preserving spread as to change people who are around-the-mean risk-averse to either less or more risk-averse. In the case of $\rho^D > \mu_\rho$, the change that makes people less risk-averse will not create many new entrepreneurs because most of them would have become entrepreneurs anyway. Some people who are made more risk-averse, however, may change their occupational choice from entrepreneurs to wage workers. The net effect in this case should be less entrepreneurs If $\rho^D < \mu_\rho$, this argument is reversed and the number of entrepreneurs will increase. The next proposition summarizes the findings obtained in the two exercises above.

Proposition 7. *(Changing the underlying distribution of risk attitude)*

(i) *If the new distribution first-order stochastically dominates the old one, there will be more entrepreneurs but the effect on the balanced growth rate is ambiguous.*

(ii) *If the new distribution is a (non-trivial) mean-preserving spread of the old one, the number of entrepreneurs will decrease if $\rho^D > \mu_\rho$ and will increase if $\rho^D < \mu_\rho$. In either case, the effect of the mean-preserving spread on the balanced growth rate is ambiguous.*

6 Calibrating the Model to U.S. Data and Quantifying Misallocation

This section describes our calibration strategy and presents quantitative evidence in favor of the potentially large misallocation losses predicted by our theory. The decentralized model economy is calibrated to post-war U.S. time series and cross-sectional establishment-level data coming from the U.S. Census Business Dynamics Statistics (BDS) for the period $1978 - 2019$. The length of one model period/generation is taken to be 25 years.[9]

6.1 Parametrization, Calibration, and Baseline Model Output

Marginal and joint densities. Each agent is characterized by a realization of the random vector (ρ, z), for which we need to determine an invariant joint distribution. A simple and transparent way to do is by first specifying each marginal density.

In line with a plethora of studies on entrepreneurship and firm dynamics, the distribution of entrepreneurial ability (z) is assumed to follow a Pareto distribution with scale parameter equal to one and shape parameter $\eta > 0$; its corresponding probability density is $g_z(z) = \eta z^{-(\eta+1)}$. In our numerical analysis we consider the finite support $[1, z_{max}]$, discretized on 600 equispaced grid points, with the upper bound set such that $G_z(z_{max}) = 0.99991$.

Regarding the distribution of risk attitude (ρ), one would naturally want to consider functions with non-negative support that result in modes/medians being towards relatively low values of ρ. A reasonable assumption is the Lognormal distribution, i.e, $\log \rho \sim \mathcal{N}(\mu_\rho, \sigma_\rho^2)$. In our numerical analysis we consider the support $[\rho_{min}, t_{max}]$, discretized on 600 equispaced grid points, with the bounds set such that $G_\rho(\rho_{min}) = 0.00001.$ and $G_\rho(\rho_{max}) = 0.95$.

To pin down the joint distribution in the baseline calibration, we simply assume that the two random variables are independent at the population level, such that the joint density of entrepreneurial ability and risk aversion becomes $g_{\rho,z}(\rho, z) = g_\rho(\rho)\, g_z(z)$. In a following subsection we examine cases where the random vector does exhibit *a priori* statistical dependence by coupling the marginals into the joint via the use of parametric copulas.

Assigned parameter. The parameter θ governing the elasticity of substitution among intermediate goods produced by entrepreneurs, given by $\sigma = \frac{1}{(1-\theta)}$, is externally calibrated. We set θ to $2/3$, or equivalently, $\sigma = 3$, which is close to the median values of σ estimated by Broda and Weinstein (2006) across 4-digit industries, as well as acroos different levels of disaggregation.[10]

Calibrated parameters. There remain seven parameters to be jointly calibrated for the model to best fit seven relevant moments in the data. In particular, the vector under consideration is $\{A, \alpha, \beta, q, \eta, \mu_\rho, \sigma_\rho\}$. Although strong local first-order identification is rather difficult in this class of non-linear general equilibrium models, the selected moments are sufficiently informative about the calibrated parameters so that the objective function is not locally flat along any direction. Below we discuss the determination and measurement of the targeted moments.

[9] Accordingly, annualized variables, e.g., real GDP growth and interest rates, are calculated as $x^{1/25} - 1$.

[10] The implied price markup, $\mu = \frac{\sigma}{\sigma-1} = \frac{1}{\theta} = 1.5$, is consistent with the estimates of De Loecker et al. (2020) for the median markup of manufacturing firms in the U.S. Censuses.

Table 1: Model Calibration to U.S. Data; Moments and Parameters

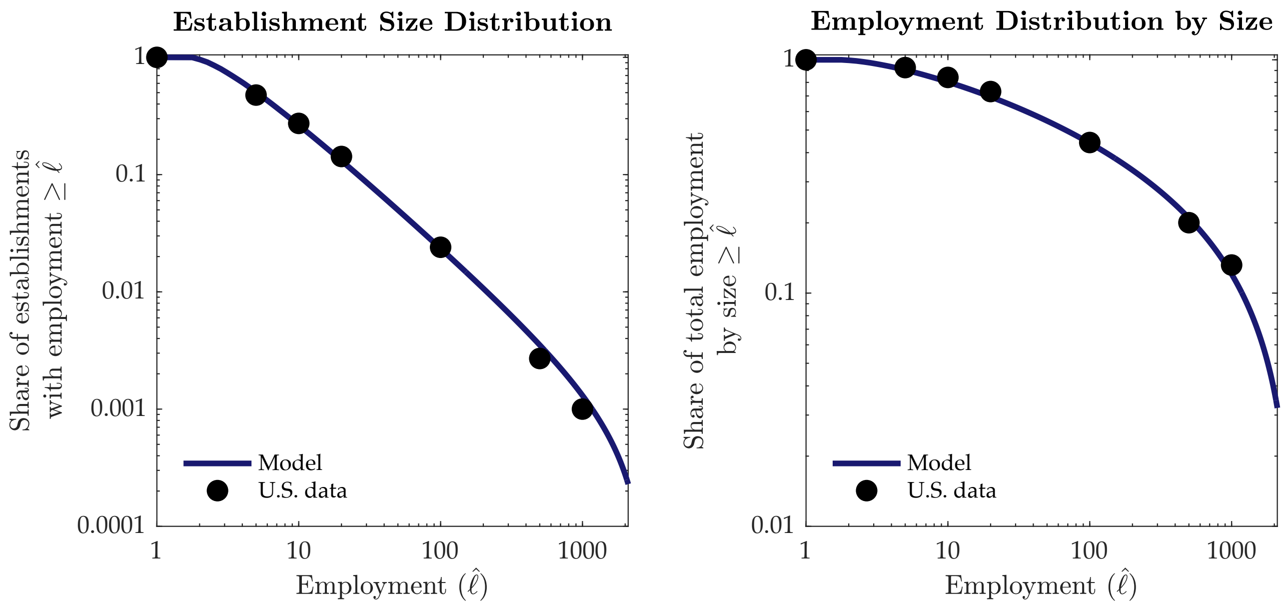
Targeted Moments	Model	Data
Annualized growth rate of real GDP per capita (g^D)	0.020	0.020
Measure of active entrepreneurs (qN^d)	0.052	0.052
Annualized real loan rate (δ)	0.035	0.035
Annualized real deposit rate (r)	0.005	0.005
Physical capital-to-output ratio	2.900	2.900
Share of establishments with $\ell \geq 100$	0.024	0.024
Employment share of establishments with $\ell \geq 100$	0.442	0.442
Calibrated Parameters	**Value**	
Productivity scaling parameter (A)	3.389	
Physical capital elasticity of output (α)	0.410	
Output elasticity of human capital (β)	0.896	
Probability of entrepreneurial success (q)	0.479	
Pareto tail parameter (η)	2.286	
Mean of $\log \rho$ (μ_ρ)	0.942	
Standard deviation of $\log \rho$ (σ_ρ)	1.911	
Elasticity of substitution $\left(\frac{1}{1-\theta}\right)$ [assigned]	3.000	

We start by describing the moment conditions used to calibrate the model's technological parameters: the productivity scaling parameter (A), the physical capital elasticity of output (α), and the output elasticity in the production of human capital (β). By observing equations (22), (24), and (25) it is clear that the parameters $\{A, \alpha, \beta\}$ are paramount in determining the balanced growth rate of real output per capita, the equilibrium loan rate, and the employed physical capital-to-output ratio. Following standard practice in the literature, we target an annualized growth rate of real GDP per capita of 2% and a (net) loan rate of 3.5% per annum; that is, $(g^D)^{1/25} = 1.020$ and $\delta^{1/25} = 1.035$. The target for the U.S. physical capital-to-output ratio is set to its long-run average of 2.9, as measured by the current-cost net stock of fixed assets in the BEA fixed assets tables divided by GDP.

Next we explain how to infer the entrepreneurial success rate. Due to the simple setting of our model, q is uniquely determined by the proportional difference between the real deposit and lending rates: $q = r_t/\delta_t$. We target a spread of 3% per annum, i.e., $r^{1/25} = 1.005$, based on the estimates of lending spreads in the novel dataset of Zimmermann (2019). Therefore, $q = 0.4793$.

The three distributional parameters—the Pareto tail parameter (η) for the distribution of z, and the mean (μ_ρ) and standard deviation (σ_ρ) for the Lognormal distribution of ρ—are primary determinants of occupational choice patterns and the distribution of factor demands. The first evident target is the measure of active entrepreneurs/producers in the labor force, which can be

Figure 2: Establishment Size and Employment Distributions: Model and Data



Notes: The source of U.S. data on establishment size and employment is the Business Dynamics Statistics $(1978-2019)$; data points correspond to sample averages. Quantities are displayed on a log scale.

deduced from the labor market clearing condition: $qN^d\bar{\ell} = 1 - N^d$ or $qN^d = q/(1+q\bar{\ell})$, where $\bar{\ell}$ corresponds to average number of employees. Using this expression along with estimates for the total number of employees and establishments in the BDS data, the sample average for $1978-2019$ is 0.052.[11] Next, relevant targets that are capable of providing further discipline come from the establishment-size and employment distributions. Two particularly instructive moments are the share of establishments with $\ell \geq \hat{\ell}$ and the employment share of establishments with $\ell \geq \hat{\ell}$, for some $\hat{\ell} > 0$ number of hired employees. Given the large concentration of entrepreneurship in small firms together with the disproportionate importance of large firms in terms of hiring, an evenhanded option is $\hat{\ell} = 100$ employees. The BDS sample averages correspond to 0.024 and 0.442, respectively. Finally, it is worth noting that varying α has little quantitative impact on occupational choices and size/employment distributions in general equilibrium, while different values of $\{A, \beta\}$ do not affect at all any of the above moments.

Table 1 reports the output of the calibration exercise and summarizes our parametrization. The model replicates the targeted moments very closely; this is achieved prudently by targeting only as many moments as parameters and through typical distributional specifications. As evidenced in Figure 2, the model is also successful in matching the full extent of U.S. establishment size and employment distributions, despite having targeted only one data point.

[11] Since annual average employment in the BDS data is mostly between 16 and 18 employees, the value of qN^d is quite insensitive to q for reasonable values of the success rate.

Table 2: Balanced Growth Equilibria vs Centralized Economy; Model Output

Model Output (U.S. Calibration)	Decentralized	Full Insurance	Planner
Annualized growth rate of real GDP per capita	0.020	0.026	0.027
Measure of active entrepreneurs/producers	0.052	0.097	0.070
Annualized real loan rate	0.035	0.042	—
Annualized real deposit rate	0.005	0.011	—
Physical capital-to-output ratio	2.900	2.467	2.450
Share of establishments with $\ell \geq 100$	0.024	0.009	0.014
Employment share of establishments with $\ell \geq 100$	0.442	0.290	0.326

6.2 Balanced Growth Equilibria vs Centralized Economy and Misallocation Losses

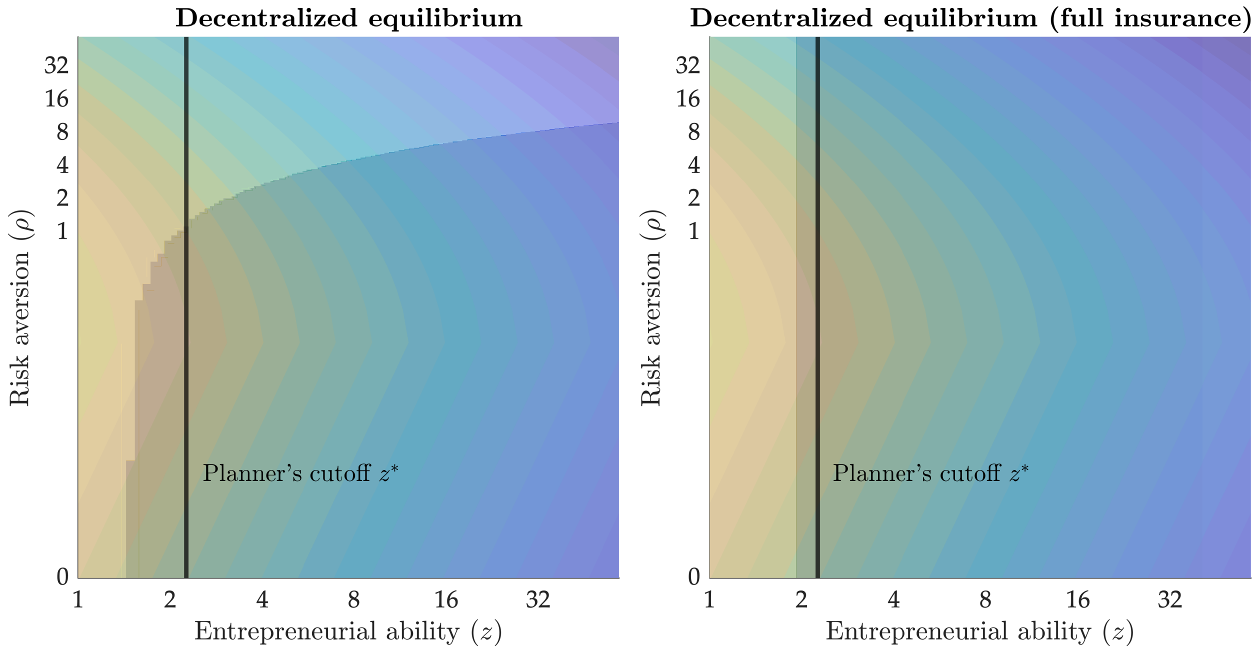
The next step is to further employ the calibrated model with a view to answering the following questions: How far from efficient are the allocations in the U.S. economy? How much of the associated losses is due to misallocation on the intensive/extensive margin? How would the competitive equilibrium change if we introduced full insurance against entrepreneurial risk?

We compute the planner's solution as well as the decentralized equilibrium with actuarially fair insurance markets and compare them to the baseline U.S. calibration. We remind the reader that while the planner is able to eradicate both types of misallocation, full insurance in the decentralized economy does eliminate misallocation on the intensive margin (risk aversion becomes irrelevant) but still features misallocation on the extensive margin ($N^{FI} > N^c$ iff $\alpha < 1/2$). Key statistics for these exercises are reported in Table 2.

The most salient points to observe are the striking gains in terms of balanced growth rates: about 0.6% on an annualized basis under full risk insurance and up to 0.7% per annum under the efficient allocations. In other words, upon removing misallocation related to occupational choices, it would take around 10 years less for real income per capita to double. The results also indicate that the U.S. entrepreneurship rate (as per our measurement) is lower than the optimal one (7%), and in the case of full insurance would lead to far too many entrepreneurs (9.7%).

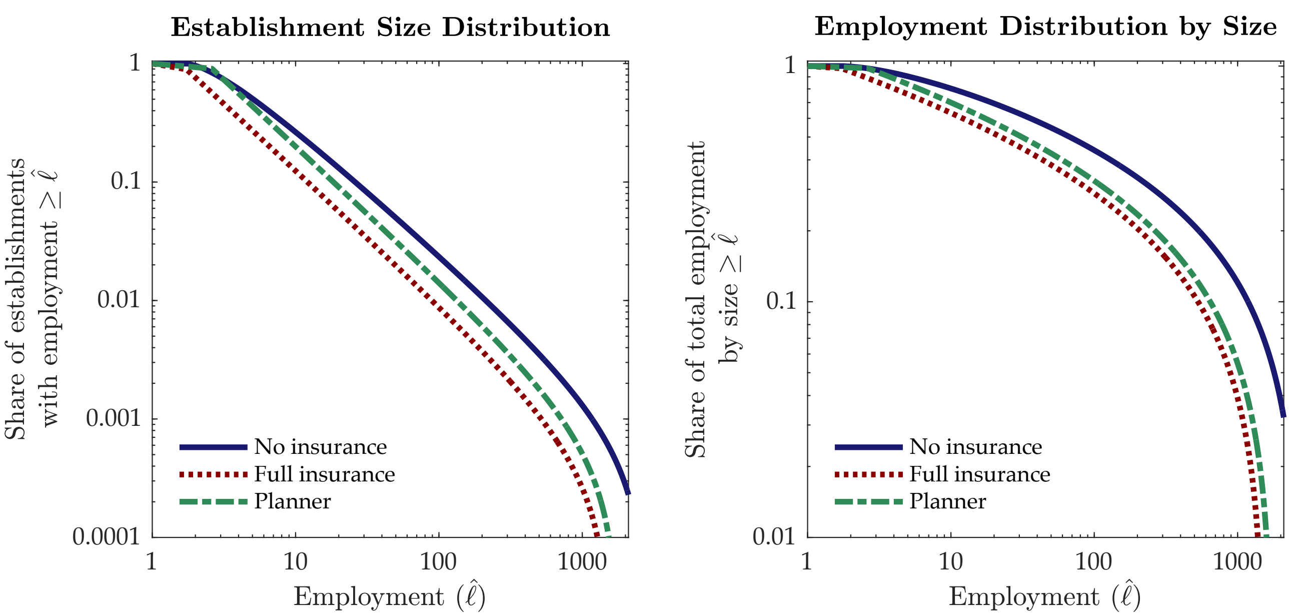
The occupational choice maps in Figure 3 offer a closer look into the anatomy of misallocation stemming from occupational sorting. Unshaded areas to the right of the planner's unique z-cutoff value represent the sizeable number of high-ability agents who do not become entrepreneurs due to high risk aversion, and the shaded areas to the left of the threshold represent a significant measure of excess entrepreneurship with lower-ability individuals. We also compute the efficient solution under the constraint that $N^c = N^d$ and find that about 97% of income growth losses (vis-à-vis the first-best) is due to misallocation on the intensive margin. That is, in the presence of misallocation due to risk aversion, *who* becomes an entrepreneur is far more important for long-run growth than *how many* people do so. A crucial policy insight is that encouraging a small number of highly skilled individuals to operate firms would be more beneficial than incentivizing a larger number of less capable entrepreneurs to do so.

Figure 3: Occupational Choice Maps: Decentralized Equilibria vs Planner



Notes: Shaded areas represent selection into entrepreneurship. The background is a filled contour plot of the joint pdf $g(\rho, z)$, with cooler colors denoting lower densities. Quantities are displayed on a log scale.

Figure 4: Comparing Establishment Size and Employment Distributions



Notes: Model solutions under the baseline parametrization. Quantities are displayed on a log scale.

7 Concluding Remarks

When it comes to promoting economic growth, is it always beneficial to encourage more entrepreneurship? Is it always desirable to have entrepreneurial risk insured away? This paper has explored the role of risk aversion and entrepreneurial ability in shaping occupational choices and balanced growth within a highly tractable endogenous growth model with heterogeneous agents. Several key insights emerge from the analysis, such as the finding that entrepreneurship and insurance provision against entrepreneurial risk may be harmful for long-run growth.

First, the relationship between the rate of entrepreneurship and balanced growth is non-monotone in general equilibrium. Increasing the number of entrepreneurs has three distinct effects on growth: a positive variety effect from expanding the range of intermediate goods, a negative loanable fund supply effect from reducing the number of workers/savers, and a TFP effect from lowering/increasing the average productivity of active firms due to occupational choices. The interplay of these forces leads to an ambiguous link between the rate of entrepreneurship and balanced growth, contrary to the conventional wisdom that ”more is more.”

Second, the decentralized equilibrium allocations are suboptimal—even without any firm-level distortions or financial frictions—and feature misallocation on both the extensive and intensive margins. Due to the presence of risk aversion, the competitive market consistently undervalues the marginal social benefits and costs of entrepreneurship, and the inverse association between risk tolerance and ability though occupational choices leads to lower aggregate TFP.

Third, introducing actuarially fair insurance markets to eliminate entrepreneurial risk in the decentralized economy does not restore the first-best allocations. While full insurance aligns private and social marginal benefits, it still fails to correct the undervaluation of marginal costs, resulting in excessive entry when the capital share is less than the labor share. Some insurance is almost always growth-enhancing, but full insurance is never optimal.

Calibrating the model to U.S. data reveals substantial output-side misallocation, with most of income growth and aggregate TFP losses stemming from the intensive margin due to risk aversion. Moreover, the U.S. entrepreneurship rate (inferred using BDS data) is lower than socially optimal; providing full insurance would result in too many, less productive entrepreneurs, but is still able to induce substantial growth gains since it eliminates intensive-margin misallocation. This suggests policies aimed at encouraging a small mass of highly talented individuals to start firms may be more effective than broad-based incentives for entrepreneurship.

All in all, even in cases where entrepreneurial entry should be encouraged to some degree, optimizing the number of entrepreneurs is not equivalent to maximizing growth. Ultimately, *what type* of individuals will choose to start firms and shape the productive capacity of an economy matters substantially more than *how many* will do so.

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A Appendix: Additional Proofs and Derivations

Proof of Proposition 2: The constrained planner solves a variational calculus problem in two steps in order to maximize the balanced growth rate, $1 + g^C = h_{t+1}/h_t = (Y_t/h_t)^{\beta}$, subject to the resource constraints. This is done by choosing feasible allocations over the set of admissible controls on the Sobolev space $\mathcal{H}^1(\Omega)$:

$$\max_{\substack{k_{t+1}(z)\ell_{t+1}(z),\\ N^c(\rho,z),\mathcal{E}^c(\rho,z)}} A^{\beta} \left(\iint\limits_{\mathcal{P}\times\mathcal{Z}} z^{\theta} \left(k_{t+1}(z)/h_{t+1}\right)^{\alpha\theta} \ell_{t+1}(z)^{(1-\alpha)\theta} \, dG(\rho, z, \mathcal{E}^c) \right)^{\frac{\beta}{\theta}} \tag{37}$$

$$\text{subject to} \iint\limits_{\mathcal{P}\times\mathcal{Z}} k_{t+1}(z) \, dG(\rho, z, \mathcal{E}^c) = \theta(1-\alpha)Y_t \tag{38}$$

$$q \iint\limits_{\mathcal{P}\times\mathcal{Z}} \ell_{t+1}(z) \, dG(\rho, z, \mathcal{E}^c) = 1 - N^c \tag{39}$$

The first step of the optimization problem involves the optimal choice of $k_{t+1}(z), \ell_{t+1}(z)$, while keeping $N^c(\rho, z), \mathcal{E}^c(\rho, z)$ fixed. Let J denote the value functional (37) and H_k, H_ℓ the constraints (38), (39) in standard form. Define the Lagrangian functional $\mathcal{L}[k(z), \ell(z)] = J - \lambda_k H_k - \lambda_\ell H_\ell$. By strict concavity and differentiability, the first-order conditions for a global maximum are necessary and sufficient,

$$\delta_\xi \mathcal{L}(k; \ell, \xi) = \frac{\partial}{\partial \varepsilon} \mathcal{L}(k + \varepsilon\xi; \ell)\Big|_{\varepsilon=0} = 0 \tag{40}$$

$$\delta_\xi \mathcal{L}(\ell; k, \xi) = \frac{\partial}{\partial \varepsilon} \mathcal{L}(\ell + \varepsilon\xi; k)\Big|_{\varepsilon=0} = 0 \tag{41}$$

where δ_ξ denotes the Gateaux derivative in the direction of ξ, for all compactly supported smooth functions ξ vanishing at $\partial\Omega$. Rearranging the FOCs and applying the fundamental lemma of the calculus of variations yields the multipliers,

$$\lambda_k = \frac{\alpha\beta(Y_t/h_t)^{\beta-1}}{\theta(1-\alpha)} \tag{42}$$

$$\lambda_\ell = \frac{(1-\alpha)\beta(Y_t/h_t)^{\beta}}{1 - N^c} \tag{43}$$

By substituting the multipliers back into the FOCs and resource constraints, one can solve the resulting system of equations to uncover the optimal entrepreneurial allocations,

$$\frac{k_{t+1}(z)}{h_{t+1}} = \left[z^\theta A^\theta (q\theta(1-\alpha))^{1-(1-\alpha)\theta} (1-N^c)^{\theta(1-\alpha)} \left(\frac{Y_t}{h_t}\right)^{(1-\beta)(1-(1-\alpha)\theta)-\theta} \right]^{\frac{1}{1-\theta}} \tag{44}$$

$$\ell_{t+1}(z) = \left[z^\theta A^\theta (q\theta(1-\alpha))^{\alpha\theta} (1-N^c)^{1-\alpha\theta} \left(\frac{Y_t}{h_t}\right)^{\theta(\alpha-\alpha\beta-1)} \right]^{\frac{1}{1-\theta}} \tag{45}$$

$$\frac{y_{t+1}(z)}{h_{t+1}} = \left[z A^\theta (q\theta(1-\alpha))^{\alpha} (1-N^c)^{1-\alpha} \left(\frac{Y_t}{h_t}\right)^{\alpha-\alpha\beta-\theta} \right]^{\frac{1}{1-\theta}} \tag{46}$$

Finally, substituting the production plan into the objective functional and manipulating through yields the social planner's long-run growth rate,

$$1 + g^C = \left[A\,(q\theta(1-\alpha))^\alpha \, \mathcal{M}_\vartheta^C \, (qN^c)^{\frac{1-\theta}{\theta}} (1-N^c)^{1-\alpha} \right]^{\beta/(1-\alpha+\alpha\beta)} \tag{47}$$

where $\mathcal{M}_\vartheta^C := \left(\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E}^C\right]\right)^{\frac{1-\theta}{\theta}}$ and the planner determines the set $\mathcal{E}^C$ so that there is no misallocation on the intensive margin from occupational sorting stemming from risk aversion.

The second step involves choosing the optimal rate of entrepreneurship, N^c, and the set $\mathcal{E}^C$. By totally differentiating $1 + g^C$ the optimality condition reads,

$$\frac{1-\theta}{\theta}\frac{1}{N^c} - \frac{1-\alpha}{1-N^c} = -\frac{1-\theta}{\theta} \frac{\frac{\partial}{\partial N^c}\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E}^c\right]}{\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E}^c\right]}$$

Next we derive an expression for the numerator on the RHS by making use of the following key observations. In the absence of misallocation, the above expression is independent of $G_\rho(\rho)$, hence the change in $\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E}\right]$ by increasing N^c is exactly equal to the change induced by decreasing the threshold $\overset{*}{z}$ adjusted by the derivative of the cdf (the pdf) at the cutoff point $\overset{*}{z}$. Also, $\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E}\right]$ is a positive and monotonic function, hence its partial derivative has the same magnitude whether we are increasing or decreasing the function w.r.t. N^c. Therefore, using the definition of truncated conditional expectation together with the Leibniz integral rule we get,

$$\frac{\partial}{\partial N^c}\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E}^c\right] = -\frac{g(\overset{*}{z})\left(\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E}^c\right] - \overset{*}{z}^{\frac{\theta}{1-\theta}}\right)}{g(\overset{*}{z})(1-G_z(\overset{*}{z}))} = -\frac{1}{N^c}\left(\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E}^c\right] - \overset{*}{z}^{\frac{\theta}{1-\theta}}\right) \tag{48}$$

Diving through by $\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E}^c\right]$ and substituting back into the optimality condition,

$$\frac{1-\theta}{\theta}\frac{\varPhi^c}{N^c} = \frac{1-\alpha}{1-N^c}$$

which yields the optimal number of entrepreneurs,

$$N^c = \frac{\varPhi^c(1-\theta)}{(1-\alpha)\theta + \varPhi^c(1-\theta)}, \quad \varPhi^c := \left(\frac{\overset{*}{z}}{\mathcal{M}^c_\vartheta}\right)^{\frac{\theta}{1-\theta}} \tag{49}$$

where optimal TFP is defined as $\mathcal{M}^c_\vartheta := \mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^c\right]^{\frac{1-\theta}{\theta}}$. ■

Proof of Proposition 3: We start by deriving an expression for N^{FI}. Recall that in the case of actuarially fair markets for entrepreneurial risk, the occupational choice condition reads

$$\frac{1-\theta}{\alpha\theta}\,\bar{z}^{\frac{\theta}{1-\theta}} = \frac{N^{FI}}{1-N^{FI}}\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^{FI}\right]$$

Since the above is independent of ρ, the cutoff $\bar{z}$ is unique and applies to every agent in the economy. We can thus easily solve for the number of entrepreneurs under full insurance:

$$N^{FI} = \frac{\varPhi^{FI}(1-\theta)}{\alpha\theta + \varPhi^{FI}(1-\theta)}, \quad \varPhi^{FI} := \left(\frac{\bar{z}}{\mathcal{M}^{FI}_\vartheta}\right)^{\frac{\theta}{1-\theta}} \tag{50}$$

with corresponding TFP defined as $\mathcal{M}^{FI}_\vartheta := \mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^{FI}\right]$. We continue by inferring N^d. Recall that in the decentralized economy there exist uncountably many occupational choice conditions:

$$q^{\hat{\rho}}\frac{1-\theta}{\alpha\theta}\,\hat{z}^{\frac{\theta}{1-\theta}} = \frac{N^d}{1-N^d}\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}\right]$$

This equation holds for all joint realizations $(\rho_i < \hat{\rho}(z_j), z_j > \hat{z}(\rho_i)) \in \mathcal{P} \times \mathcal{Z}$, thus N^d can be pinned down from an infinite number of pairs. Define the set $\mathcal{F} := \{\varPhi^D_j := \hat{z}_j^{\frac{\theta}{1-\theta}}/\mathbb{E}[z^{\frac{\theta}{1-\theta}}|\mathcal{E}] : \rho_i < \hat{\rho}(z_j), z_j > \hat{z}(\rho_i)) \in \mathcal{P} \times \mathcal{Z}\}$. In general, the number of entrepreneurs satisfies:

$$N^d = \frac{q^{\hat{\rho}}\varPhi^d_j(1-\theta)}{\alpha\theta + \varPhi^d_j(1-\theta)}, \quad \varPhi^d_j := \left(\frac{\hat{z}}{\mathcal{M}_\vartheta}\right)^{\frac{\theta}{1-\theta}} \tag{51}$$

We now seek appropriate values for $\varPhi^D_j$ to ease the comparison with N^{FI} and examine the two cases that can arise depending on distributional assumptions.

Case 1: $\exists \varPhi^D_j = \varPhi^{FI} \in \mathcal{F}$. In this case it is straightforward to conclude that $N^{FI} > N^d$ $(a.s.)$, since $q^{\hat{\rho}} < 1$ given that $\tilde{\rho} > 0$.

Case 2: $\nexists \varPhi^D_j = \varPhi^{FI} \notin \mathcal{F}$. This case may apply if and only if $\min(\hat{z}_j) \geq \bar{z}$. Instead of examining $\varPhi^D_j$, it is easier to consider the simple fact that under full insurance occupational choice is independent of risk aversion. Hence, there exists at least one measurable set such that $\mathcal{S} = \{(z = \min(\hat{z}_j)) \wedge (\rho \in \mathcal{P})\} \subset \mathcal{E}^{FI}$ and $\mathcal{S} \cap \mathcal{E} = \varnothing$. The set has positive measure and $\mathcal{P}(\mathcal{E}^{FI}) \geq \mathcal{P}(\mathcal{E} \cup \mathcal{S})$, which implies that $N^{FI} > N^d$ $(a.s.)$. ■.

Intermediate Lemma: The proof is immediate once we identify a crucial insight. Although the number of entrepreneurs under full insurance markets may differ from the centralized economy, in both scenarios there is no misallocation on the intensive margin, as occupational choices are independent of ρ and the z-cutoff is unique (but not necessarily the same). One could think of the resulting occupational choice sets as forming two *similar rectangles*, defined on the measurable space $(\Omega, \mathcal{B}(\Omega))$ equipped with the push-forward probability measure $\mathbb{P}_{T,Z}$.

For this reason the endogenous TFP term – a function of a truncated conditional expectation – will be (conditionally) maximal in both cases, and since the lower truncation point is respectively determined by a single z-cutoff, this translates to:

$$\frac{\bar{z}}{\mathcal{M}_{\vartheta}^{FI}} = \frac{\overset{*}{z}}{\mathcal{M}_{\vartheta}^{c}}$$

$$\left(\frac{\bar{z}}{\mathcal{M}_{\vartheta}^{FI}}\right)^{\frac{\theta}{1-\theta}} =: \Phi^{FI} = \Phi^{c} := \left(\frac{\overset{*}{z}}{\mathcal{M}_{\vartheta}^{c}}\right)^{\frac{\theta}{1-\theta}} \tag{52}$$

■

Proof of Proposition 4: The proof is straightforward given the result of the intermediate lemma above. Rearrange (50) and (49) to get,

$$\alpha = \frac{\Phi^{FI}(1-\theta)(1-N^{FI})}{\theta N^{FI}}$$

$$1-\alpha = \frac{\Phi^{c}(1-\theta)(1-N^{c})}{\theta N^{c}}$$

Since $\Phi^{FI} = \Phi^{c} = \Phi \in \mathbb{R}_{+}$, this proves that $N^{FI} = N^{c}$ if and only if $\alpha = \frac{1}{2}$. As both equations are strictly decreasing in N, we can conclude directly that $N^{FI} > N^{c} \iff \alpha < \frac{1}{2}$ and $N^{FI} < N^{c} \iff \alpha > \frac{1}{2}$. ■