Econ 508B: Introduction to Measure, Probability, and Econometric Theory

Washington University in St. Louis – Summer 2023

Instructor:	Alexandros Loukas	**Synchronous Lectures:**	M–F 2pm-4pm (CST)
Email:	alex.loukas@wustl.edu	**Classroom:**	Seigle Hall, Room 301
Course Page:	Canvas webpage	**Zoom ID:**	Zoom meetings

Communication and Office Hours

- You can always reach out to me by email about any questions you may have. The official turnaround time for emails is 24h, but I will typically respond to you faster.
- I will be holding unofficial office hours after the end of each lecture, in addition to online or in-person meetings by appointment.

Course Description and Learning Objectives

Econ 508B is an intensive three-week preparatory class for incoming doctoral students in economics, finance, and other related disciplines. This course is designed to equip students with a working knowledge of reasonably rigorous (yet intuitive) probability theory using measure theory, and to introduce several fundamental tools of statistics and econometrics. As such, it will be of both theoretical and practical value to all Ph.D. students who focus on micro, macro, finance, econometrics, or any other field with a significant quantitative orientation.

The main objectives of the course are twofold. The short-run goals are to cover the prerequisites for the first-year Ph.D. courses "Applied Econometrics" and "Quantitative Methods in Economics," and at the same time to deepen your understanding of various concepts and tools that will be used in other first-year core courses. The long-run goal is to help you develop one very essential skill: the ability to tackle problems involving stochastic objects in a mathematically precise way. This will not only enable you to better understand other people's research, but will also facilitate your own needs towards more theoretically sound and meaningful research.

Class Attendance and Participation

Regular attendance is essential and expected. Student participation and collaboration are highly encouraged in and out of the classroom, and will greatly benefit your learning experience. All lectures will be recorded and uploaded on Canvas in order to facilitate your review process.

Grading Policy

The final grade for this course will be a weighted average of the items listed below. You are strongly encouraged to consult with each other and collaborate by forming study groups. However, each student must turn in their own work – you have to be prepared for what is coming up this year.

- Problem Set 1 (30%)
- Problem Set 2 (30%)
- Final Exam (40%)

Tentative Schedule and Lecture Outline

- **Lectures 1–2**

  Review of Set Theory; Algebras and σ-Algebras of events, Borel sets, Measurable spaces; Measure, Probability measure; Kolmogorov Axioms of Probability; Conditional Probability, Bayes' Theorem, Independence; Foundations of Zero-One Laws.

- **Lectures 3–4**

  The Problem of Measure and the Need for Measure Theory; Construction of Lebesgue measure; Carathédory Construction of Measures; Completion of Measures; Measurable Functions.

- **Lectures 5–6**

  Lebesgue Integration; Comparison between the Lebesgue and the Riemann Integral; Key Convergence Theorems; Decomposition of Measures; Product Measure and Repeated Integration.

- **Lectures 7–8**

  Random Variables; Induced Probability Measures, Distribution Functions, Density Functions; Independence of Random Variables; Transformation of Random Variables; Expectation, Variance, and Covariance; Moment Generating Functions; Characteristic Functions.

- **Lectures 9–10**

  Conditional Expectation; Law of Iterated Expectations, Conditioning Theorems, Law of Total Variance; The CEF Decomposition Property and Regression Fundamentals; The Linear CEF Model, Best Linear Predictor.

- **Lecture 11**

  Probabilistic Inequalities; Markov's Inequality, Chebyshev's Inequality, Chernoff Bounds; More Fundamental Inequalities: Jensen, Cauchy-Schwarz, Hölder, Minkowski; Stochastic Orders, Asymptotic Notation.

- **Lecture 12**

  Modes of Stochastic Convergence and their Relations; Continuous Mapping Theorem, Slutsky's Theorem; Laws of Large Numbers; Central Limit Theorems; Delta Method.

- **Lectures 13–14**

  The Theory of Least Squares Estimation; The Algebra of OLS; The Geometry of OLS; Generalized Least Squares; Generalized Gauss-Markov Theorem; Asymptotic Properties of OLS.

- **Lecture 15**

  Model Estimation; Maximum Likelihood Estimation; Generalized Method of Moments.

Useful References

The course has been designed to be largely self-contained, with a large collection of lecture slides available on our Canvas page. Please note that slides may be modified and updated along the way.

Nevertheless, I have decided to include a non-exhaustive list of references based on both personal favorites and widely revered textbooks/monographs. I do believe you will find them illuminating and particularly helpful, not only during the course of the first and second year of your Ph.D. but also throughout your academic and professional career.

Measure and Probability Theory

Ash, Robert B., and Doléans-Dade, Catherine A. (2000), *Probability and Measure Theory*, 2nd edition, Harcourt/Academic Press.

Billingsley, Patrick (1995), *Probability and Measure*, 3rd edition, John Wiley and Sons.

Fristedt, Bert, and Gray, Lawrence F. (1997), *A Modern Approach to Probability Theory*, Birkhauser.

Pollard, David (2001), *A User's Guide to Measure Theoretic Probability*, Cambridge Series in Statistical and Probabilistic Mathematics, Cambridge University Press.

Rosenthal, Jeffrey S. (2006), *A First Look at Rigorous Probability Theory*, 2nd edition, World Scientific Publishing Co.

William, David (1991), *Probability with Martingales*, 1st edition, Cambridge University Press.

Statistics

Casella, George, and Berger, Roger (2001), *Statistical Inference*, 2nd edition, Cengage Learning.

Hogg, Robert, McKean, Joseph, and Craig, Allen (2012), *Introduction to Mathematical Statistics*, 7th edition, Pearson Education.

Van der Vaart, A. W. (2000), *Asymptotic Statistics*, Cambridge University Press.

Wasserman, Larry (2004), *All of Statistics: A Concise Course in Statistical Inference*, Springer.

Econometrics

Angrist, Joshua D., and Pischke, Jörn-Steffen (2009), *Mostly Harmless Econometrics: An Empiricist's Companion*, Princeton University Press.

Davidson, Russell, and McKinnon, James G. (2014), *Econometric Theory and Methods*, Oxford University Press.

Goldberger, Arthur S. (1991), *A Course in Econometrics*, Harvard University Press.

Greene, Wiliiam H. (2018), *Econometric Analysis*, Pearson, New York.

Hansen, Bruce E. (2022), *Econometrics*, Princeton University Press.

Hayashi, Fumio (2000), *Econometrics*, Princeton University Press.

White, Halbert (2014), *Asymptotic Theory for Econometricians*, Academic Press.